<u>Chaotic Motion of a Double Pendulum</u>

Background

Many real world systems exhibit chaotic behavior. Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions. Diverging outcomes for chaotic systems due to differences in initial conditions render long term prediction to become impossible in general. However, the systems future behavior is fully determined by their initial conditions with no random elements involved. One example is the idea that a butterfly can influence the weather on the other side of the globe. A very small atmospheric perturbation can give rise to large effects due to the nonlinear nature of the system.

The double pendulum is designed to be the one of the rare simple devices that demonstrates true chaotic behavior. It typically consists of a bar swinging from a pivot, and the second pendulum attached to the first bar’s end. Furthermore, the deterministic nature of these systems does not make the double pendulum unpredictable.

Objective

Regarding the double pendulum as an example of a chaotic system, we try to predict its motion. Aim of this research is to demonstrate that predicting long-term motion in a chaotic system is difficult to almost impossible.

Materials

The material list for the construction of the double pendulum consists of nine materials. For the main pivot on which the pendulum rotates, an 8mm steel rod of 3’’ length is used. To reduce friction to an absolute minimum and to get the best performance, high quality skateboard bearings are used. Other necessary components to build the pendulum include: 5/16’’ bolts, 1’’ long, with nuts (2), a sheet of polycarbonate plastic 12’’x12’’x1/4’’ thick, a 3V coin cell battery, an optional LED for time-lapsed photography, rubber washers 1’’ ID (2), nylon washers or plastic spacers, and finally, an optional 8mm shaft collar. Data recording materials include a protractor, fishing rod, and timer.

Method

To test the chaotic theory of a double pendulum, a double pendulum must be first constructed. Once the process is complete, three persons are needed to gather data. One person will be the timer, while another will count the number of flips. The third person will record the data.

To initiate the process, the angles of the first and second pendulum must be measured by a fishing line and protractor. The procedure consists of a set of forty-nine measurements of for different angles. The angle difference between two measurements is fifteen degrees in one of the two angles. In the initial settings the angles are zero degrees.

The first pendulum is labeled Pendulum A and the second Pendulum is labeled Pendulum B. Pendulum A will begin with one angle degree measure that will remain the same while Pendulum B will have different angle measurements differing fifteen degrees until the last angle measurement is 90 degrees. This process will continue until both pendulums are measured at 90 degrees. For each set of measurements a time interval of ten, thirty, and sixty seconds is used. One person will count the number of flips the pendulum makes within each time interval.

Results

The trajectories of the double pendulum are continuous and complicated, as illustrated in Figure 1.

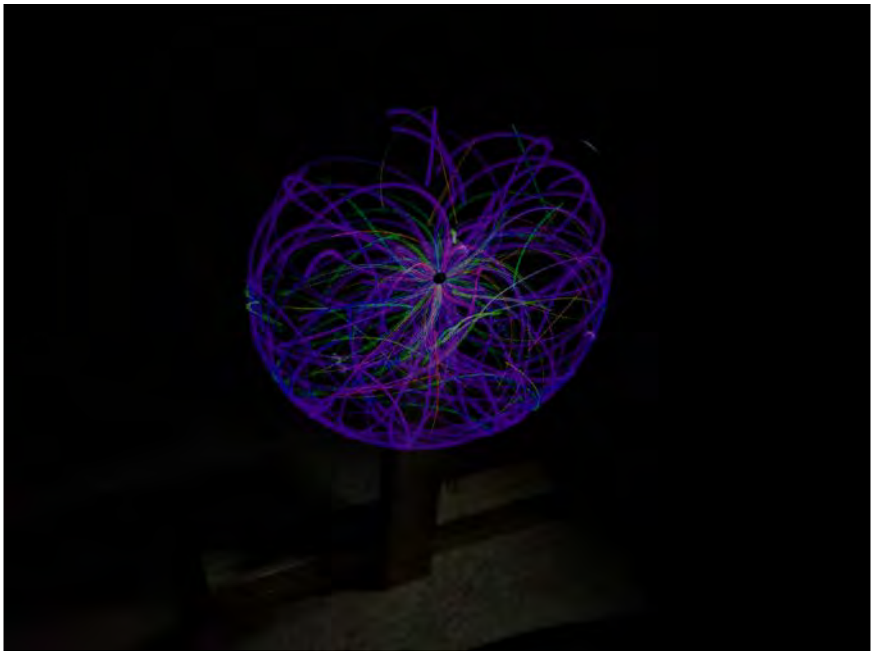

Figure 1: Trajectory of a double pendulum for one particular choice of initial parameters.

To determine if the dynamics of the double pendulum can be predicted, the number of flips within a specified time interval was counted. Figure 2 shows the number of flips as a function of the initial angles of pendulum A and pendulum B.

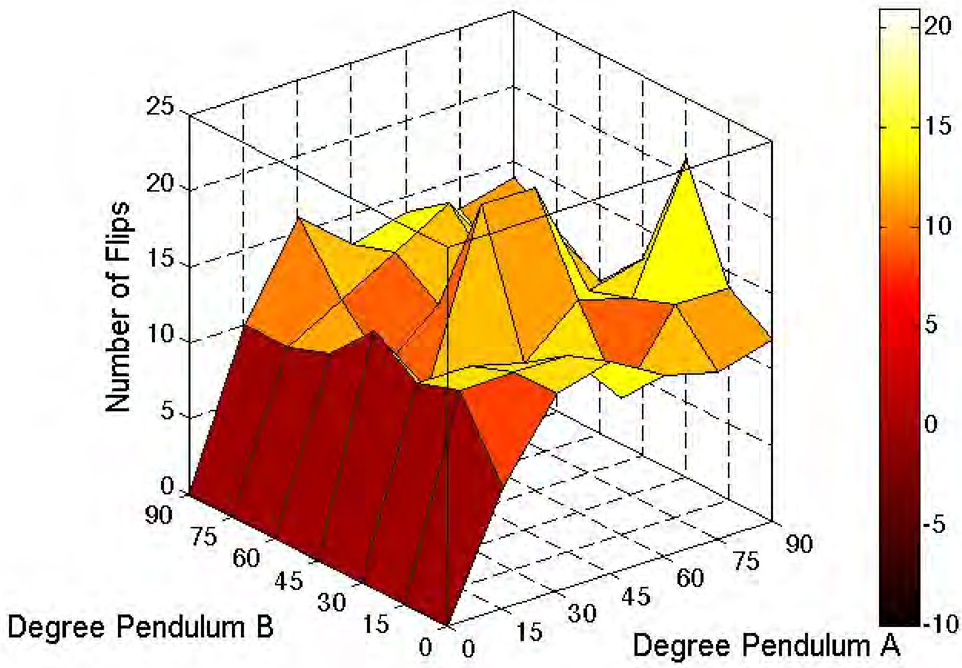

Figure 2: Number of flips within 10 seconds.

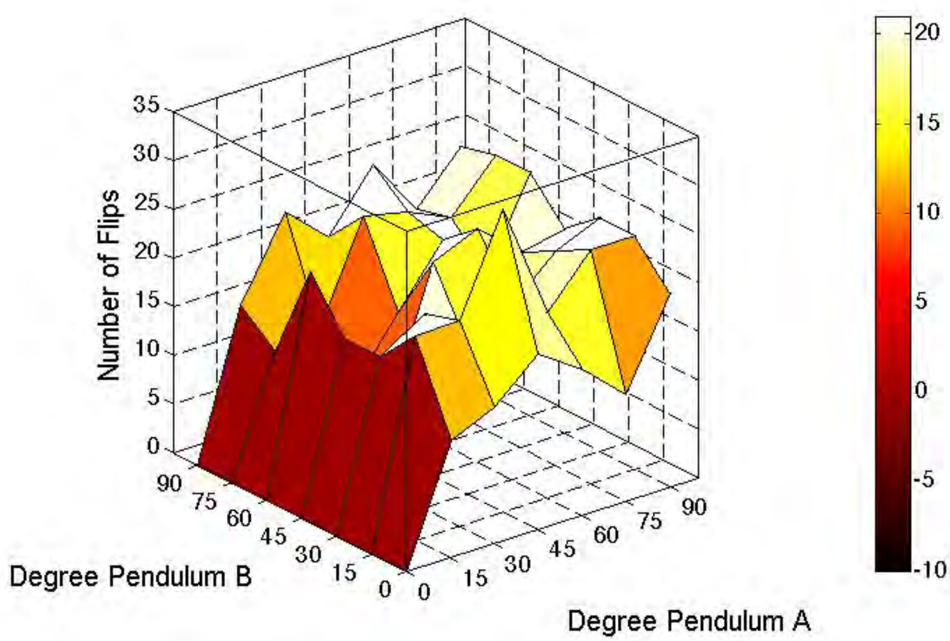

Figure 3: Number of flips within 30 seconds.

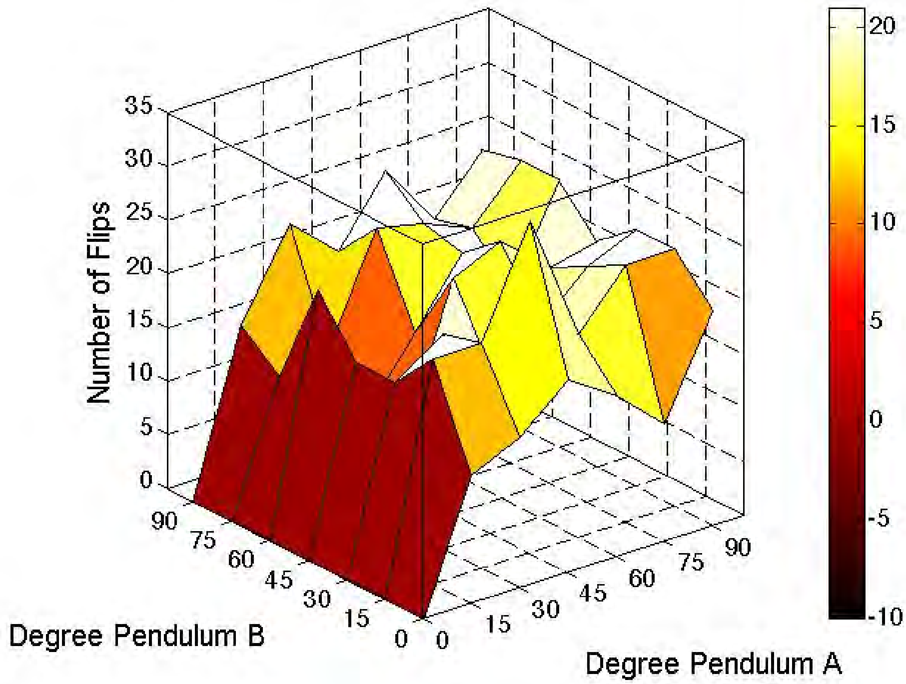

Figure 4: Number of flips within 60 seconds.

Figure 4 illustrates the number of flips between 10 and 30 seconds.

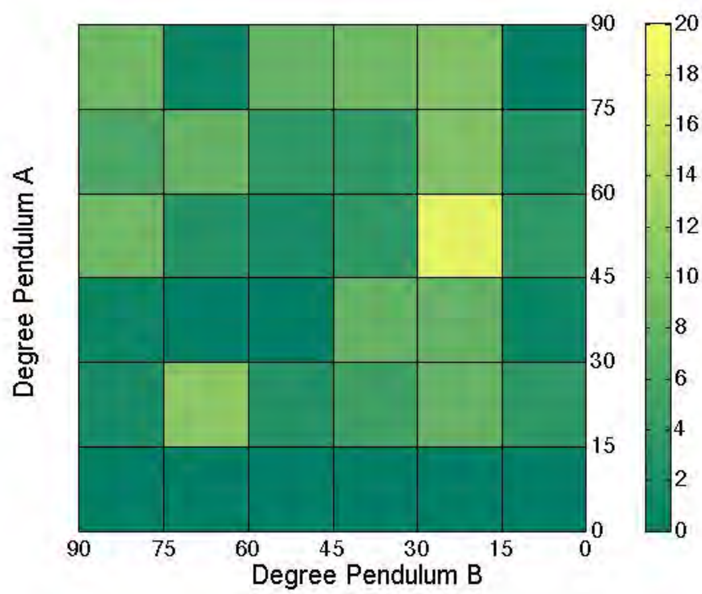

Figure 5: Number of flips between 10 and 30 seconds.

Analysis

Figure 2, 3, and 4 do not indicate a functional relationship between the two initial angles of the double pendulum and the number of observed flips. One simple case can be identified: if the top pendulum has an initial degree of zero degrees, no flips are observed for all angles of the second pendulum. For initial angles greater than zero no patterns or relationships between the number of flips for different initial positions were found. Since the double pendulum obeys Newtonian dynamics, its behavior is deterministic and no random elements contribute to its trajectory. Therefore, our results indicate sensitive dependence of the number of flips on both initial angles. We found no relationship between the number of flips between 10 and 30 seconds and the initial angles. This confirms our finding that the dynamics of the pendulum critically depends on the initial conditions.

Conclusion

The double pendulum is a conceptually simple system that exhibits fascinating and rich dynamics. Although a single pendulum can easily be described mathematically, we showed that it is impossible to predict the dynamics of the double pendulum.

The concept of chaotic behavior is common to many real world systems. The interaction of simple entities can result in unpredictable behavior of the system as a whole. This concept can be expanded to biological and social science and is a frontier of 21st century science. How do ant colonies find the optimal food paths? How does the interaction between neurons results in consciousness? Chaos theory is a possible approach to find answers to these questions.

Acknowledgments

Chaotic Motion of a Double Pendulum

Dat Ly, Taaha Akhtar, Robert Xavier Neiberding
KIPP Houston High School
Houston TX 77099

Theoretical Background

- The double pendulum is designed to be the one of the rare devices that demonstrates true chaotic behavior. It typically consists of a bar swinging from a pivot, and the second pendulum attached to the first bar's end. Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions.

- Furthermore, the deterministic nature of these systems does not make the double pendulum unpredictable.

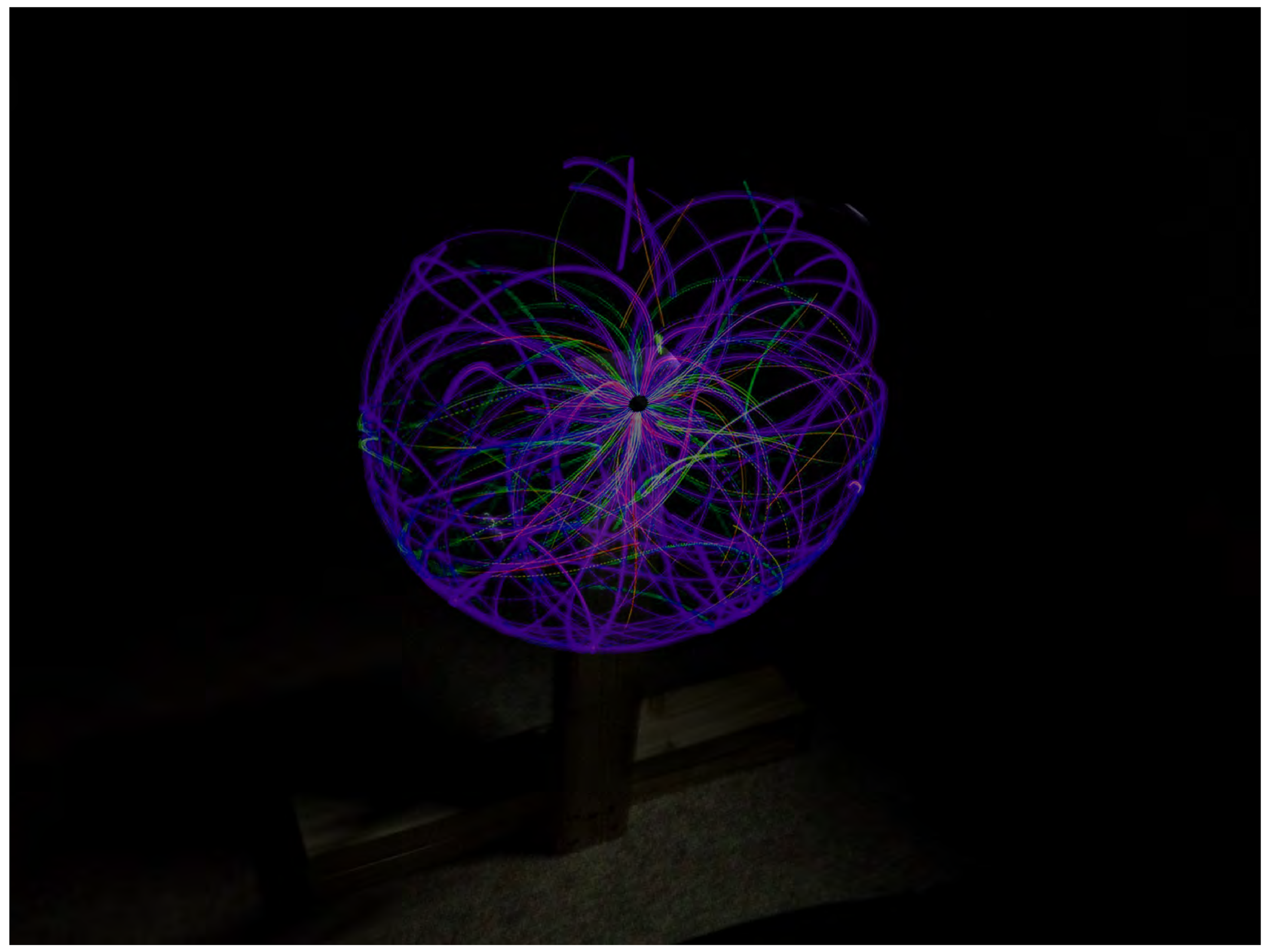

Figure 1: Trajectory of a double pendulum for one particular choice of initial parameters.

Objectives

- Regarding the double pendulum as an example of a chaotic system, we try to predict its motion.

- Aim of this research is to demonstrate that predicting long-term motion in a chaotic system is difficult to almost impossible.

Materials

- The material list for the construction of the double pendulum consists of nine materials. For the main pivot on which the pendulum rotates, an 8mm steel rod of 3'' length is used. To reduce friction to an absolute minimum and to get the best performance, high quality skateboard bearings are used.

- Other necessary components to build the pendulum include: 5/16'' bolts, 1'' long, with nuts (2), a sheet of polycarbonate plastic 12''x12''x1/4'' thick, a 3V coin cell battery, an optional LED for time-lapsed photography, rubber washers 1'' ID (2), nylon washers or plastic spacers, and finally, an optional 8mm shaft collar. Data recording materials include a protractor, fishing line, and timer.

Procedures

- To initiate the process, the angles of the first and second pendulum must be measured by a fishing line and protractor. The procedure will begin with a total set of forty-nine measurements of fifteen degree angles. The pendulums will start with an initial angle of zero degrees.

- The first pendulum is labeled as Pendulum A and the second Pendulum is labeled as Pendulum B. Pendulum A will begin with one angle degree measure that will remain the same while Pendulum B will have different angle measurements of fifteen degrees until the last angle measurement is 90 degrees. This process will continue until both pendulums are measured at 90 degrees. For each set of measurements a time interval of ten, thirty, and sixty seconds is used. One person will count the number of flips the pendulum makes until sixty seconds.

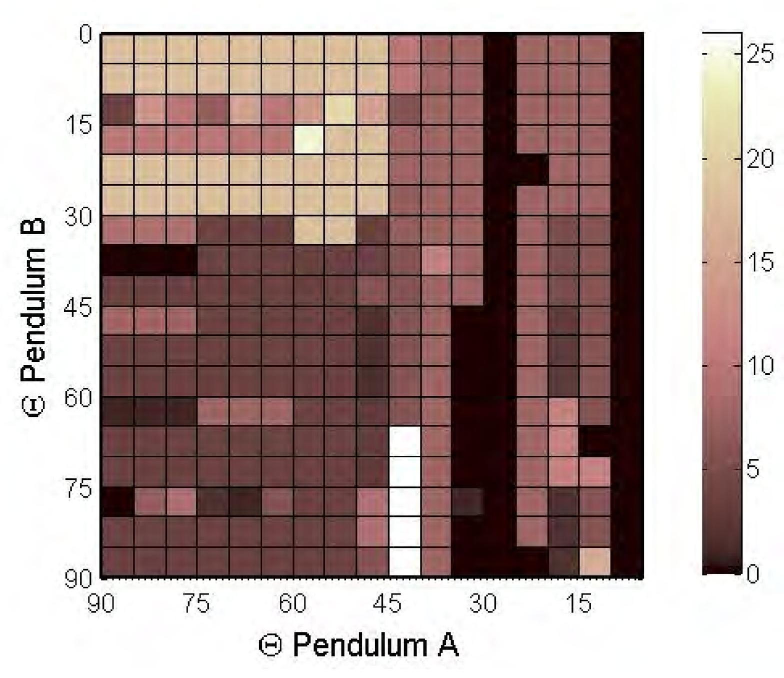

Figure 2: Number of flips between 10 and 30 seconds.

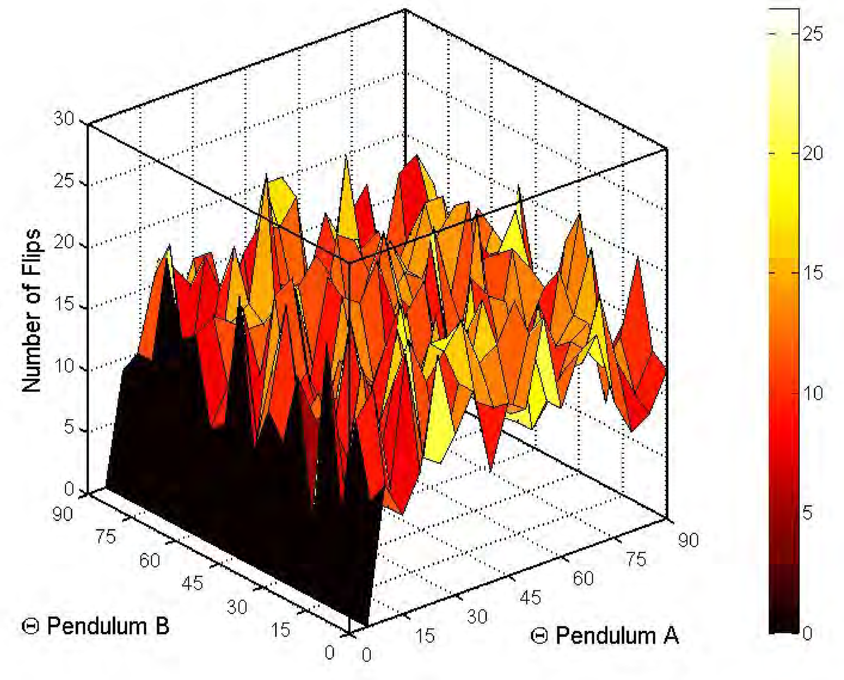

Figure 3: Number of flips within 10 seconds.

Results and Analysis

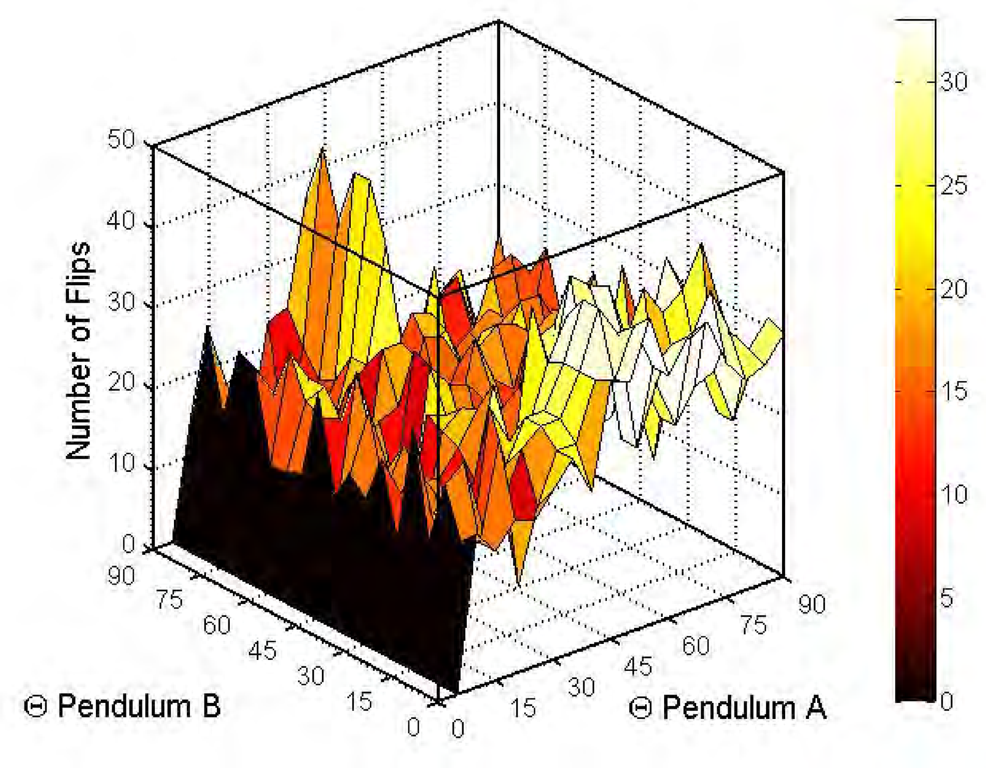

Figure 4: Number of flips within 30 seconds.

- Figure 2, 3, and 4 do not indicate a functional relationship between the two initial angles of the double pendulum and the number of observed flips.

- No relationship between the number of flips between 10 and 30 seconds and the initial angles can be found. This confirms our finding that the dynamics of the pendulum critically depends on the initial conditions

Conclusion

- Although a single pendulum can easily be described mathematically, it is impossible to predict the dynamics of the double pendulum.

Acknowledgments

- The researcher would like to thank Ms. Mila Bersabal, for her valuable input and guidance in this study.

- He would like to thank Mr. Seemann for helping the researcher to learn more about chaotic theory and double pendulums. He also would like to thank Thanh Lam and Raul Recinos for helping build the double pendulum.